THE ALVEOLAR EQUATION

(Derivation from First Principles)

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An attempt is here made to derive, from first principles, the Alveolar Equation:—

$$pO_2 = FO_2 (P_B - 47) - pco_2 \left(\frac{1-FO_2 \ (1-RQ)}{RQ} \right)$$

2. In Fig. 1, let:—

(a) 'x' be the volume of "physiological alveolar" air in the lungs at the end of expiration. (By "physiological" is meant that portion of the gas mixtures in the respiratory apparatus that takes part in gaseous exchange).

(b) 'd' be the volume of "physiological dead space" air in contact with mucous membrane of the respiratory apparatus and hence saturated with water vapour but not taking part in other respiratory gas exchange.

(c) 'd + a' be the increase in volume of the gases in the respiratory system at the end of inspiration. This increase consists of two parts; a portion 'd' which is the water saturated dead space air drawn in during inspiration and a portion 'a' derived from outside air. 'd + a' is in fact the tidal volume.

(d) 'P_B' be the ambient atmospharic pressure.

3. Then 'x + d + a' is the volume of the "physiological alveolar" air at the end of inspiration, assuming no gaseous interchange has yet taken place. If it is assumed that the outside air is dry then the volume of inspired air is $\frac{P_B - 47}{P_B}(a + d)$, to which is added $\frac{47}{P_B}(a + d)$ of water vapour.

$\frac{P_B - 47}{P_B}(a + d)$ = volume of dry outside air inspired.

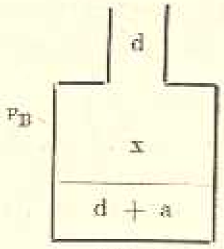

Fig. 1.

x = volume of "physiological alveolar" air at the end of expiration.

x+d+a = volume at the end of inspiration.

4. If 's' is the volume of CO_2 (dry) given out by the lungs during each respiratory cycle and 'z' is the volume of dry oxygen taken in, then:—

(a) 's – z' is the volume of the "output" of dry gases into the alveolar system.

(b) A further $\frac{47\ (s-z)}{P_B - 47}$ of H_2O is added to this to saturate it.

(c) $\frac{s}{z}$ = RQ (The respiratory quotient)

5. After respiratory gaseous exchange, the volume 'v' of the "physiological" alveolar air therefore becomes:—

$$v = x+d+a+s-z+\frac{47}{P_B - 47}\ (s-z)$$

or $$v = x+d+a+\frac{P_B\ (s-z)}{P_B - 47}$$ ....Equation (1)

6. In a "Steady State" of respiration, the composition of this air at the end of gaseous exchange, will be the same as the composition of the volume 'x' at the commencement of inspiration.

7. Of the volume $x + d + a + \frac{P_B\ (s-z)}{P_B - 47}$, the various gases occupy:—

(a) Oxygen = $f.x + k.d + \frac{P_B - 47}{P_B}\ a.o - z$ where f, k and o are fractions of oxygen in alveolar, dead space and atmospheric air respectively.

Note:— Of outside air $\frac{P_B - 47}{P_B}$ (a + d) only, the portion $\frac{P_B - 47}{P_B}$ a reaches the alveoli.

(b) Nitrogen = $g.x + l.d + \frac{(P_B - 47)}{P_B}\ a.p.$ where g, l and p are corresponding fractions of nitrogen.

(c) Carbon dioxide = $h.x + m.d + s$ where h, m and NIL are corresponding fractions for CO_2.

(d) Water vapour = $j.x + n.d + \frac{47}{P_B}\ a + \frac{47}{P_B - 47}(s-z)$ where j, n and NIL are the fractions for H_2O.

Note:— Atmospheric air is assumed to be dry and contain no carbon dioxide.

8. The alveolar fractions, f, g, h and j would, in the "Steady State" of respiration be equal to the fractions (a), (b), (c) and (d) of para 7 divid ed by the volume 'v' of Equation (1) i.e.

$$f = \frac{f.x + k.d + \frac{(P_B - 47)}{P_B}\ a.o - z}{x + d + a + \frac{P_B\ (s-z)}{P_B - 47}}$$ ....Equation (2)

$$g = \frac{g.x + l.d + \frac{P_B - 47)}{P_B}\ a.p}{x + d + a + \frac{P_B\ (s-z)}{P_B - 47}}$$ ....Equation (3)

$$h = \frac{h.x + m.d + s}{x + d + s + \dfrac{P_B\,(s - z)}{P_B - 47}} \qquad \text{...............Equation (4)}$$

9. From Equations (2), (3) and (4) we have:—

$$f.x + f.d + f.a + f.\,\frac{P_B\,(s - z)}{P_B - 47} = f.x + k.d + \frac{(P_B - 47)}{P_B}\,a.o - z \;\text{...........Equation (5)}$$

$$g.x + g.d + g.a + g.\,\frac{P_B\,(s - z)}{P_B - 47} = g.x + l.d + \frac{(P_B - 47)}{P_B}\,a.p \;\text{...............Equation (6)}$$

and $$h.x + h.d + h.a + h.\,\frac{P_B\,(s - z)}{P_B - 47} = h.x + m.d + s \;\text{...............Equation (7)}$$

10. In Equation (5) f.d. = k.d. because 'd' at the end of expiration can be assumed to have the same composition as 'x', the volume 'd' being smaller than 'd' + 'a'. Similarly g.d. = l.d. and h.d. = m.d. Hence :—

$$f\left[a + \frac{P_B\,(s - z)}{P_B - 47}\right] = \frac{(P_B - 47)}{P_B}\,a.\,o - z \qquad \text{...............Equation (8)}$$

$$g\left[a + \frac{P_B\,(s - z)}{P_B - 47}\right] = \frac{(P_B - 47)}{P_B}\,a.p \qquad \text{...............Equation (9)}$$

and $$h\left[a + \frac{P_B\,(s - z)}{P_B - 47}\right] = s. \qquad \text{...............Equation (10)}$$

11. Dividing Equation (8) by Equation (10) we have:—

$$\frac{f}{h} = \frac{(P_B - 47)\,a.o - P_B.\,z}{P_B.\,s} \qquad \text{...............Equation (11)}$$

12. Dividing Equation (9) by Equation (10) we have:—

$$\frac{g}{h} = \frac{(P_B - 47)\,a.p}{P_B.\,s} \qquad \text{...............Equation (12)}$$

13. By definition g, (the fraction of nitrogen in alveolar air) is $1 - \frac{47}{P_B} - f - h.$, and from Equation (12), $a = \frac{P_B.gs}{(P_B - 47)\ h.p}$

Equation (12) may therefore be written:—

$$a = \frac{P_B.\,s\left(1 - \dfrac{47}{P_B} - f - h\right)}{(P_B - 47)\,h.p} \qquad \text{...............Equation (13)}$$

14. Also by definition p, the fraction of N_2 in outside air, is 1 − o; Equation (13) therefore becomes:—

$$a = \frac{P_B.\,s\left(1 - \dfrac{47}{P_B} - f - h\right)}{(P_B - 47)\,h.\,(1 - o)} \qquad \text{...............Equation (14)}$$

15. Substituting the value of 'a' of Equation (14) in Equation (11), we have:—

$$\frac{f}{h} = \frac{\dfrac{(P_B - 47).o.[P_B.s(1 - \frac{47}{P_B} - f - h)]}{(P_B - 47).h.(1 - o)} - P_B.z}{P_B.s}$$

$$\therefore f = \frac{o(1 - \frac{47}{P_B} - f - h)s}{s(1 - o)} - \frac{h.z(1 - o)}{s(1 - o)}$$

$$\therefore f.s - s.f.o = o.s. - \frac{47\ o.s.}{P_B} - s.f.o - s.h.o - h.z + h.z.o$$

$$\therefore f = o - \frac{47.\ o}{P_B} - h.o. - \frac{h.z}{s} + \frac{h.z.o}{s} \qquad \text{Equation (15)}$$

16. If each term of Equation (15) is multiplied by P_B and $\frac{1}{RQ}$ is substituted for $\frac{z}{s}$, we have:-

$$P_B\ .\ f = P_B.o - 47.o - P_B.h.o - \frac{P_B.h}{RQ} + \frac{P_B.h.o}{RQ}$$

$$\text{or} \quad P_B\ .\ f = (P_B - 47).\ o - P_B.h\ (o + \frac{1}{RQ} - \frac{o}{RQ}) \qquad \text{Equation (16)}$$

17. Now, f is the volume fraction of alveolar oxygen, hence $P_B.f$ = the pressure fraction. i.e. pO_2 (the alveolar oxygen tension). Similarly $P_B.h$ is alveolar CO_2 tension.

o is volume fraction of oxygen in atmospheric (breathed) air, i.e. FO_2. Substituing in Equation (16) we have:—

$$pO_2 = FO_2\ (P_B - 47) - pCO_2\ (FO_2 + \frac{1}{RQ} - \frac{FO_2}{RQ})$$

or

$$pO_2 = FO_2\ (P_B - 47) - pCO_2\ (\frac{1 - FO_2\ (1 - RQ)}{RQ})$$

18. **Conclusion**

It is felt that the above method of deriving the Alveolar Equation gives a good insight into the assumptions, such as the "Steady State" on which the equation is based.